**Recent Development of Fuzzy Regression Model**

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**Abstract:** The objective of this paper is to illustrate the recent results in modelling fuzzy regression. First, the fuzzy regression model proposed by H. Tanaka is illustrate and then I introduce fuzzy AR model. Based on this model I propose the fuzzy switching AR model. In order to analyze natural wards, I build linguistic regression model.

**1. Fuzzy Regression Model**

As for a fuzzy multivariate analysis based on the extension principle, a fuzzy linear regression model, a fuzzy time-series model, a possibility linear model, and so on have already been formulated by H. Tanaka [7] and J. Watada [8] and so on. The fuzzy linear regression analysis aims to build a model that expresses possibilities which a system has. As all samples realize a possibility which the system should have, the fuzzy regression model is formulated so as to include all samples in itself. Therefore, it is also named a possibility regression model.

A fuzzy regression analysis is to model a possibilistic structure using given data, that is, to evaluate possibilities of the system which are embodied in data. Therefore, it is also named a possibilistic regression model. In the sense of possibility, a possibilistic regression analysis is built to include whole samples in the fuzzy model.

The built fuzzy regression is formulated as follows:

$$ Y = A_1 x_1 + A_2 x_2 + \cdots + A_n x_n = A x $$

where $x_1 = 1$, partial regression coefficient $A_i$ is a triangular fuzzy number denoted as $A_i = (a_i, c_i)$ with its center $a_i$ and its width $c_i$ and $A = (a, c)$.

A fuzzy coefficient with center $a$ and width $c$, and

\[ a = [a_1, a_2, \ldots, a_n] \]
\[ c = [c_1, c_2, \ldots, c_n] \]
\[ x' = [x_1, x_2, \ldots, x_n] \] (2)

As $Ax = (a, c)x = (ax, c|x|$ by extension principle, the output of the fuzzy regression shown in Equation (1) is also a fuzzy number, where $|x'| = |x_1|, |x_2|, \ldots, |x_n|$. According to this operation, Equation (1) can be written as:

$$ Y = Ax = (ax, c|x|) $$

The fuzzy regression model with partial fuzzy coefficients has lower limit $ax - c|x|$ and upper limit $ax + c|x|$. When observed data $(y_j, x_j)(j = 1, 2, \ldots, m)$ which consist of $Y_j = (y_j, e_j)$, $x'_j = [x_{1j}, x_{2j}, \ldots, x_{nj}]$ are obtained, the inclusion relation between the model and data is written as follows:

$$ Y_j \in Ax_j = (ax_j, c|x_j|) $$

That is, 

$$ ax_j + c|x_j| \geq y_j + e_j $$
$$ ax_j - c|x_j| \leq y_j - e_j $$

(5)

In other words, the possibilistic regression analysis is constructed to include whole samples in the fuzzy regression model. But the larger the width of the model which is constructed to include whole samples is, the vaguer the model becomes, because the meaning of the model is not clear. As a result, the expression of the regression becomes uncertain. It is required to build the regression model with so narrow width as possible, so that the vagueness of the model can be removed. Therefore, the estimation of a possibilistic regression model can result in the following linear programming by which we can find a model with the smallest width out of feasible models which satisfy the constraints (5) of whole samples:

\[
\begin{align*}
\text{minimize} & \quad \sum_{j=1}^{m} c|x_j| \\
\text{subject to} & \quad y_j \leq ax_j + L^{-1}(\alpha)c|x_j| \\
& \quad y_j \geq ax_j - L^{-1}(\alpha)c|x_j| \\
& \quad c \geq 0 \quad (j = 1, 2, \cdots, m)
\end{align*}
\]
2. Fuzzy AR Model

It is necessary to employ some analysis and data that we interpret economic structure. Many econometric models including time-series model have been proposed for this purpose. In this paper, we proposed the fuzzy time-series model which is employed in analyzing an economic system. The objective of the economic analysis is to understand the past and present states of the economic system more precisely based on the statistical data. However, in the economic system, it is closely related to many factors which bring out by the aggregation of many human behaviors. Therefore, it is insufficient to interpret such an economic system only based on the results of conventional statistical method. So, it is desirable to apply the concept of the fuzzy system theory which can cope with the ambiguity of a structure when we analyze the economic system which has many vague factors.

We analyze Nikkei stock average employing the fuzzy autoregressive model which proposed by Ozawa et al [2] and fuzzy autocorrelation model which is proposed in this paper. Furthermore, it is expectable that we can forecast a future trend by sequential prediction which fits the present condition.

2.1. Fuzzy Autocorrelation Model

In this section, we explain the fuzzy autoregressive model which is proposed by Ozawa et al and the fuzzy autocorrelation model which is proposed in this paper. Though a fuzzy autoregressive model employs a trapezoidal fuzzy number in [2], we employ a triangular fuzzy number in our model. Therefore, we explain a fuzzy autoregressive model in terms of triangular fuzzy numbers.

In the fuzzy autocorrelation model, time-series data \( z_t \) are transformed into a fuzzy number to express the possibility of data. The following fuzzy equation shows the case where only one time point before and after time point \( t \) is taken into consideration in building a fuzzy number [14][8].

\[
Y_t = (Y_t^L, Y_t^C, Y_t^U) = (\min(z_{t-1}, z_t, z_{t+1}), z_t, \max(z_{t-1}, z_t, z_{t+1}))
\]

Next, we employ a calculus of finite differences to filter out the time-series data of trend. It enables us first-order difference-equation to write the following:

\[
T_t = (T_t^L, T_t^C, T_t^U) = (\min(Y_t - Y_{t-1}, Y_t^C - Y_{t-1}^C, \max(Y_t - Y_{t-1}), z_t, \max(z_{t-1}, z_t, z_{t+1}))
\]

Generally, if we take finite differences then we reduce the trend variation, and only an irregular pattern is included in the difference series. However, when we calculate the fuzzy operation, the ambiguity may become large and the value of an autocorrelation coefficient may also take 1 or more and -1 or less value. In order to solve this problem in the case of the fuzzy operation, we adjust the width of a fuzzy number using \( \alpha \)-cut when we calculate the difference series. An \( \alpha \)-cut level \( h \) is decided from the value of the autocorrelation. When we calculate the fuzzy autocorrelation, we employ usual fuzzy operation under condition that the fuzzy autocorrelation of lag 0 is set \( \rho_0 = \lambda_0/\lambda_0 = (1,1,1) \). It results in the following linear programming to decide the value at the \( \alpha \)-cut level.

When we set \( \alpha \)-cut level to 1, the ambiguity of fuzzy autocorrelation is made smallest, but we cannot obtain the fuzzy autocorrelation which reflects the possibility of the system. So, we maximize the width of autocorrelation. However, the size of width is decided automatically as the value of autocorrelation should be include in [-1,1].

\[
\begin{align*}
\text{maximize} & \quad \sum (\rho_i^U - \rho_i^L) \\
\text{subject to} & \quad \rho_i^L \leq 1 \\
& \quad \rho_i^L \geq -1 \\
& \quad \rho_i^L \leq \rho_i^C \leq \rho_i^U \\
& \quad (i = 1, 2, \ldots, p)
\end{align*}
\]

We can define the fuzzy covariance and the fuzzy autocorrelation as follows:

\[
\begin{align*}
\Lambda_k & = \text{Cov}[T_t T_{t-k}] \\
& = E[T_t T_{t-k}] = [\lambda_k^L, \lambda_k^C, \lambda_k^U] \\
r_k & = \Lambda_k/\Lambda_0 = [\rho_k^L, \rho_k^C, \rho_k^U]
\end{align*}
\]

We adjust the ambiguity of the difference series employing the \( \alpha \)-cut level \( h \) which is obtained by solving the above linear programming. Using fuzzy autocorrelation coefficient which is calculated by employing \( \alpha \)-cut level \( h \), we redefine Yule-Walker equations as in linear programming, and calculate the partial autocorrelation \( \pi \).

We calculate the following autoregressive process.

\[
T_t = \Phi_1 T_{t-1} + \Phi_2 T_{t-2} + \cdots + \Phi_p T_{t-p}
\]

where \( \Phi = [\phi^L, \phi^C, \phi^U] \) is a fuzzy partial autoregressive coefficient.

As mentioned above, the next value of observation value exceeds observed value at present by the size of the value of autocorrelation, or it is less. For this reason, autocorrelation is important for the time-series analysis. So, we build the model which illustrates ambiguity of the system shown by fuzzy autocorrelation. The reason for the autocorrelation is also fuzzy auto-
correlation, Yule-Walker equations can be also calculated by the fuzzy equation in the same way.

\[ R_t = \Phi_1 r_{t-1} + \Phi_2 r_{t-2} + \cdots + \Phi_p r_{t-p} \]  

(8)

\( \Phi \) in Equation (8) is an unknown coefficient. We are building the model in terms of fuzzy autocorrelation which can describe the ambiguity of the system. However, when ambiguity of a model is large, the relation between a model and a system becomes ambiguous. Therefore, the possibility of the system can not be described properly. So, in order to obtain the fuzzy partial autocorrelation coefficient whose ambiguity of a time-series model should be minimized, we come down to the following linear programming:

\[
\begin{align*}
\text{minimize} & \quad \sum (\rho_t^U - \rho_t^L) \\
\text{subject to} & \quad R_t^U \geq \rho_t^U \\
& \quad R_t^C = \rho_t^C \\
& \quad R_t^L \leq \rho_t^L \\
& \quad \rho_t^L \leq \rho_t^C \leq \rho_t^U \\
& \quad (t = 1, 2, \cdots, p)
\end{align*}
\]

As mentioned above \( R \) is obtained by the fuzzy operation employing fuzzy autocorrelation \( r \) and fuzzy partial autocorrelation \( \Phi \). \( R^L, R^C, R^U \) represent the lower limit, the center, and the upper limit of \( R \), respectively.

A fuzzy autocorrelation model expresses the possibility that the change of the system is realized in data, different from the conventional statistical method. We are building the model which can show an ambiguous portion called a possibility that it has not expressed clearly by the conventional statistics technique.

2.2. Numerical Example

In this section, we employ Nikkei stock average which indicates the trend of the whole stock market as an index of Japanese stock market. We use the monthly data from 1970 to 1998.

We show the sample autocorrelation coefficient at each time lag (Figure 2) in order to determine the order. Figure 2 shows the negative correlation in lags 1 and 2 where the sign of the autocorrelation coefficient changes minus to plus. Because of this result, we analyze Nikkei stock average employing AR(2) model with the second-order.

Furthermore, because of existing seasonal variation in this data, we employ the calculation \( \nabla^2 \nabla_{12} \tilde{z}_t \) which take the first-order seasonal difference of every 12 period after processing taking the second-order difference.

\[ \nabla^2 \nabla_{12} \tilde{z}_t = \phi_1 \nabla^2 \nabla_{12} z_{t-1} \]
Figure 3: Fuzzy Autocorrelation

Figure 4: The conjectured result by Fuzzy Autocorrelation Model

\[ \phi_2 \nabla^2 \nabla_{12} Z_t - 2 + u \]

where the data \( z_t \) in analysis are statistical data and actual measurement. \( u \) is an error term of the model.

In order that ambiguity of time-series system reflects on these data, we employ these data to fuzzy numbers to deal with these data.

Next, we analyze Nikkei stock average employing the fuzzy autocorrelation model which is proposed in this paper. In this model, we set the \( \alpha \)-cut level 0.088. The fuzzy autocorrelation showed minus correlation in lag 2 similar to the case of the autocorrelation.

In the estimated fuzzy autocorrelation model, the coefficient was determined as follows:

\[ \nabla^2 \nabla_{12} \tilde{Z}_t = (-0.834, -0.642, -0.000) \nabla^2 \nabla_{12} Z_{t-1} + (-1.00, -0.380, -0.380) \nabla^2 \nabla_{12} Z_{t-2} \]

The model which is obtained by the fuzzy autocorrelation model has a negative coefficient the same as the result which is obtained by the fuzzy autoregressive model.

Original series and estimated series are shown in the Figure 4.

Figure 5: The prediction result in 1999 by Fuzzy Autocorrelation Model

As shown in Figure 4, the estimated model has small width and has brought the small fuzziness. Numerically, the width of the possibility of the model is 2,500 yen on the average, 18,000 yen in the maximum and 100 yen at the minimum.

3. Fuzzy Switching AR Model

It is necessary to analyze data in order that we interpret an economic structure. Many econometric models including time-series analysis have been proposed for this purpose.

When we analyze a system employing a time-series model, it is difficult to use a regression model because of complexity of systems. This fact is also the same in the case where the characteristic of system should be changed.

In these cases, it is necessary to separate data according to a decision of analyzer.

When data are obtained from several systems, it is hard to correctly analyze the data by a forecasting model such as a time-series analysis without separating data according to the latent systems and analyzing separately each of the systems.

Generally speaking, it is hard to separate out of data from each system in the case where data is multivariate.

In this paper, we propose the fuzzy switching autoregression model which divide samples into each system. We analyze Nikkei stock average employing the fuzzy switching AR model as a numerical example.

3.1. Switching Regression Model

First, we illustrate switching regression model. J. C. Bezdeck[1] and Y. Nakamori[3] propose switching regression modeling, respectively. In this paper we illustrate a switching regression model according to a
Bezdek’s model.

Let us denote a set of m samples by $S$.

\[ S = \{(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)\} \]

\[ x_j = [x_{j1}, x_{j2}, \ldots, x_{jn}] (j = 1, 2, \ldots, m) \]

If we analyze data set $S$ based on the conventional regression analysis as coming from one system when data set $S$ consists of samples observed from cl systems, we cannot obtain the proper result by this analysis.

In this case, it should be necessary to separately analyze each of cl systems. The fuzzy c-regression model is to analyze data set $S$ observed from cl systems and to build a regression for each of systems separately.

Let us denote a membership grade as $u_{ij}$ to which extent cluster of data $D_j = (x_j, y_j)$, includes sample $i$, then $u_{ij}$ has the following features:

\[ 0 \leq u_{ij} \leq 1 \]
\[ 0 \leq \sum_{j=1}^{cl} u_{ij} \leq m \]
\[ \sum_{i=1}^{m} u_{ij} = 1 \]

where, $U = [u_{ij}]$.

Let us denote a multi-regression model for cluster $i$ as follows:

\[ y = f_i(x; \beta_i) + e_i \tag{9} \]

where $\beta_i$ and $e_i$ denote a partial regression coefficient and a residual of multi-regression $f_i$.

Let us denote multi-regression $f_i$ as:

\[ f_i(x; \beta_i) = x_1 \beta_{i1} + x_2 \beta_{i2} + \cdots x_n \beta_{in} \tag{10} \]

and each of variables as the following matrix form:

\[ X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}, Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}, D_i = \begin{bmatrix} u_{i1} & 0 & \cdots & 0 \\ 0 & u_{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & u_{im} \end{bmatrix} \]

Partial regression coefficient of $i$th cluster, $\beta_i$, is obtained by the following:

\[ \beta_i = (X'D_iX)^{-1}X'D_iY \]

Let us denote an error between the multi-regression (10) and given data $D_j$ as $E_{ij}$. Objective function of fuzzy c-regression model, $Z$, is written using parameter $r(r > 1)$ as follows:

\[ Z = \sum_{i=1}^{cl} \sum_{j=1}^{m} u_{ij}^r E_{ij} \tag{11} \]

The fuzzy c-regression model is to cluster samples using data set in order that the objective function(11) can be minimized by a fuzzy clustering analysis(c-means method).

### 3.2 Fuzzy Switching AR Model

Time-series data about the economy sometimes show the change which was different from the tendency until then according to the society conditions. The time-series data of economies changes its trend according to the society conditions such as the economic conditions, the influence of other countries, the war and so on. They often occur unexpectedly in most cases. Therefore, the system has different tendency because of changing trend of data. That cannot say that actual data are shown precisely about judging all data one time-series data and analyzing it.

In this paper, we employ a switching method to divide data of each system and employ a fuzzy autocorrelation model to apply for each of divided data. We divided a data according to possibility of observed system in order to illustrate possibility of model using a fuzzy time-series model. We have to grasp a possibility of being intermingled and contained in data, and have to divide each into the possibility of a system.

Therefore, in this paper, we employ a genetic algorithm to separate data into each of systems. An individual has a gene with a grade to which extent it is included in each of systems, these fitness are calculated by the width of the possibility of the same system.

Using the width of possibility of the $i$th distribution $w_i^{(j)}$, $J_i$, which is fitness of individual $i$, is as follows:

\[ J_i(w) = \sum_{i=1}^{cl} w_i^{(j)} \tag{12} \]

According to above mention, we employ a fuzzy autocorrelation model to structure a switching model for fuzzy AR model. Using the evaluation function(12), we separate data to each of systems by a genetic algorithm. We apply a fuzzy autocorrelation model to each of divided data.

### 4 Linguistic Regression Model

As Professor L. A. Zadeh has placed the stress on the importance of the computation with words,
fuzzy sets can take a central role in handling words [18, 19]. In this perspective fuzzy logic approach is often thought as the main and only useful tool to deal with human words. In this paper we intend to present another approach to handle human words instead of fuzzy reasoning. That is, fuzzy regression analysis enables us treat the computation with words.

We intend to abstract the latent structure under the relations between words. Human words can be translated into fuzzy sets such as fuzzy numbers, which is employed in a fuzzy controller. If it is possible such as fuzzy control to formulate a dictionary between fuzzy number and a word, we can build the relations under the data in terms of fuzzy regression analysis.

Consequently, only experts with much professional experiences are capable of making assessment using their intuition and experiences. The measurements and interpretation of these characteristics are taken with uncertainty, because most measured characteristics, analytical results, and field data can be interpreted only intuitively by experts. In such cases, judgments may be expressed by experts with linguistic terms. The difficulty in the direct measurement of certain characteristics makes the estimation of these characteristics imprecise. Such measurements may be dealt with the use of fuzzy set theory [4, 16, 17].

Watada, Fu and Yao [9, 10] proposed a model of damage assessment by using the information given by experts through fuzzy multivariate analysis.

In order to process linguistic variables, we define the vocabulary translation and vocabulary matching which convert linguistic expressions into membership functions on the interval [0-1] on the basis of a linguistic dictionary, and vice versa. We employ Fuzzy Regression Analysis [4, 7] in order to deal with the assessment process [13, 15] of experts’ from linguistic variables of features and characteristics of an objective into the linguistic expression of the total assessment.

4.1. Linguistic Variable and Vocabulary Matching

In making assessments, experts often (a) evaluate various features and characteristics, and (b) assess the objective in a linguistic form. For instance, though it is possible to measure the production volume, it is difficult to analytically interpret the numerical value in terms of its influence of this amount on the future decision making.

On the other hand, experts may be able to express (a) the effect of a given sales trend as "good" or "not good", and (b) the total assessment or state in a linguistic form such as “very good”, “good” or “not good” as shown in Table 1.
Consider the simplest possible value such as production volume. Let \( L \) be a linguistic variable which can have values in the set \( X \). As examples of such a variable, let us consider a production volume as well as the number of visiting customers.

\[
L_{\text{volume}}(\text{production}) \text{ is } /\text{extremely bad}/
\]

\[
L_{\text{number}}(\text{visiting customers}) \text{ is } /\text{bad}/
\]

where subscripts (number) in \( L(\text{visiting customers}) \) denote the state of a visiting customers.

These expressions “good”, “bad”, “extremely bad” can be defined with fuzzy grades on \([0,1]\) such as \( U(\text{good}), U(\text{bad}), U(\text{extremely bad}) \). Denote \( \pi \) as a possibility distribution. We can identify the possibility of the state of a sales trend with the degree of its descriptive adjective on \([0,1]\). For example,

\[
\pi(\text{number})(\text{visiting customer}) \equiv \pi(\text{state})(\text{the number}) \equiv U(\text{extremely bad})
\]

Define the dictionary of descriptive adjectives in which a descriptive adjective corresponds with its fuzzy grade on \([0,1]\).

The objective of this study is to model the experts’ assessment process through which experts might evaluate the possibility of sales trend \( L \) on the basis of states of its features and characteristics \( L_i(i = 1, 2, \ldots, K) \), where these values \( L \) and \( L_i \) are expressed in a linguistic form. In other words, we intend to determine the linguistic assessment process \( F \) of linguistic variable \( L_1, L_2, \ldots, L_K \), which produces a linguistic value of an objective \( Z \). It can be written in the form

\[
Z = F(L_1, L_2, \ldots, L_K)
\]

We define the descriptive adjectives “extreme”, “very”, and so on. Let us make a dictionary for corresponding linguistic expressions \( L_i \) and fuzzy grades \( U_{L_i} \). If we obtain the assessment \( L_i(i = 1, 2, \ldots, K) \), \( L_i \) is understood in terms of \( U_{L_i} \) through the dictionary build by experts. Let us divide this linguistic assessment process \( F \) into three portion:

1. translation of attributes form linguistic values \( L_i \) into fuzzy grades \( U_{L_i} \)

\[
U_{L_i} \equiv (u_i, c_i^l, c_i^r)
\]

where \( u_i \) denotes the central value with grade 1, \( c_i^l \) left side fuzziness and \( c_i^r \) right side fuzziness, respectively.

2. estimation of total damage by the fuzzy assessment function

\[
V = f(U_{L_1}, U_{L_2}, \ldots, U_{L_K})
\]

which produces a fuzzy grade \( V \) in terms of fuzzy grades of attributes \( U_{L_i} \) where a suffix \( L \) is not attached to \( V \) because it is unknown, and the detail of this calculation is explained in Section 4.2.

3. linguistic matching of the fuzzy grade of the objective with the dictionary wherein the linguistic value \( Z \) is decided for the objective.

Let us define the vocabulary matching by using the following minimax calculation

\[
Z_0 \simeq \max_{W_i \in D} \left\{ \max_{t} \mu_V(t) \bigwedge \mu_{W_i}(t) \right\}
\]

where \( Z_0 \simeq \max_{W_i \in D} f(W_i) \) denotes that \( Z_0 \) is the word in \( D \) which realizes the maximum value of \( f \), \( \mu_V \) denotes a membership function of \( V \), and \( \mu_{W_i} \) denotes a membership function of a word \( W_i \) as included in the dictionary \( D \). This procedure is illustrated in Figure 7. We assign a word \( L_0 \) “very bad” to the fuzzy grade of the total assessment \( V \) showed a dotted line in Figure 8.

<table>
<thead>
<tr>
<th>Training Sample</th>
<th>Linguistic variables</th>
<th>Linguistic objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( L_1(1) ) \ldots ( L_1(1) ) \ldots ( L_K(1) )</td>
<td>( L_0(1) )</td>
</tr>
<tr>
<td>2</td>
<td>( L_1(2) ) \ldots ( L_1(2) ) \ldots ( L_K(2) )</td>
<td>( L_0(2) )</td>
</tr>
<tr>
<td>3</td>
<td>“good” \ldots “bad” \ldots “very bad”</td>
<td>( \text{“bad”} )</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots \ldots \vdots \ldots \vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>\omega</td>
<td>( L_1(\omega) ) \ldots ( L_1(\omega) ) \ldots ( L_K(\omega) )</td>
<td>( L_0(\omega) )</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots \ldots \vdots \ldots \vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>( n )</td>
<td>( L_1(n) ) \ldots ( L_1(n) ) \ldots ( L_K(n) )</td>
<td>( L_0(n) )</td>
</tr>
</tbody>
</table>

Table 1: Linguistic data given by experts.


4.2 Determination of Fuzzy Regression Model

After the model of experts' assessment process is formulated, it is desirable to determine the fuzzy assessment function $f$ of the total assessment as shown in Figure 8. This is a fuzzy function with which $K$ fuzzy grades of attributes, $U_{L_i}$, can be transformed to one fuzzy grade of its total assessment, $V$. We must determine this fuzzy regression model on the basis of training data $\omega(\omega = 1, 2, \ldots, n)$ as given by experts in order to mimic experts' assessment process. Let us employ the method proposed as Fuzzy Quantification Theory Type 1 by Watada, Tanaka and Asai [7] for determining this fuzzy regression model $f$.

Table 1 shows the training data given in linguistic form by experts. This data in Table 1 is translated into data of fuzzy grades in terms of the dictionary (see Table 2). In Table 2, the fuzzy grades of attributes $i$ and of the total damage of sample structures $\omega$ are denoted by $U_{L_i}(\omega)$ and $V_{L_0}(\omega)$, respectively where $i = 1, 2, \ldots, K$ and $\omega = 1, 2, \ldots, n$. We should note that the model is estimated using given linguistic data $L_0, L_1, \ldots, L_K$.

Assume that all fuzzy grades have triangular shapes which are normal and convex. We employ a fuzzy linear function for a fuzzy regression model $f$ as

$$V_{L_0} = f(U_{L_1}, U_{L_2}, \ldots, U_{L_K})$$

$$= \sum_{i=1}^{K} A_i U_{L_i}(\omega) \quad \omega = 1, 2, \ldots, n \quad (16)$$

It is noted that $V_{L_0}$ is given value instead of $V$ is estimation.

Using $n$ relation of training samples

$$V_{L_0}(\omega) = \sum_{i=1}^{K} A_i U_{L_i}(\omega) \quad \omega = 1, 2, \ldots, n \quad (17)$$
We must specify the best fuzzy parameters $A_i$ in terms of these relations. Two criteria are employed in order to define the goodness of the fuzzy linear function. One criterion is fitness of the fuzzy regression model, $h$, and the other is fuzziness included in the fuzzy regression model, $S$.

(i) Fitness.

Assumed that an estimated value $V_{L_o}(\omega)$ is obtained by the fuzzy linear function $f$, fitness $b(\omega)$ of $Y_0(\omega)$ to a sample value $Y(\omega)$ is $V_{L_o}(\omega)$ to a sample value $V_{L_o}(\omega)$ is defined by

$$h(\omega) = \bigvee_{y \in R} \left\{ \mu_{L_o}(y) \bigwedge \mu_{L_o}(y) \right\} \quad (18)$$

(ii) Fuzziness.

The fuzziness $S^\alpha$ included in the fuzzy function at $\alpha$-level is defined by

$$S^\alpha = \sum_{i=1}^{K} (\overline{a}_i - a_i) \quad (19)$$

where $\overline{a}_i$ and $a_i$ are numbers which specify an $\alpha$-level set $A_i^\alpha$, i.e.,

$$A_i^\alpha = [\overline{a}_i, a_i] \quad (20)$$

In this paper, we deal with triangular fuzzy numbers. Note that

$$A_i^\alpha = [a_i - (1 - \alpha)c_i, a_i + (1 - \alpha)c_i]$$

as $A_i^\alpha$ is defined by

$$A_i = (a_i, c_i, c_i)$$

Note that these two indices of fuzziness and fitness are incompatible with each other. The higher fitness we seek, the resulting model will be.

(1) Formulation of the problem.

We formulate a fuzzy assessment function by minimizing its fuzziness $S$ under the constraints that an estimated fuzzy grade of structural total damage of each sample is fit to the fuzzy grade given by experts with the fitness greater than or equal to the given value $h^0$, called fitness standard.

[Problem]

If data are given such as that listed in Table 2, the problem is to determine a fuzzy linear function

$$V_{L_o}(\omega) = \sum_{i=1}^{K} A_i \cdot U_{L_i}(\omega) \quad (21)$$

which minimizes the fuzziness

$$S = \sum_{i=1}^{K} (\overline{a}_i - a_i) \quad (22)$$

under the conditions that

$$h(\omega) = \bigvee_{y \in R} \left\{ \mu_{L_o}(y) \bigwedge \mu_{L_o}(y) \right\} \geq h^0 \quad (23)$$

$$\omega = 1, 2, \ldots, n$$

where $h(\omega)$ indicates the fitness of the estimated value with respect to a sample $\omega$ and $h^0$ denotes the fitness standard, and $\overline{a}_i$ and $a_i$ are defined as

$$A_i^K = [\overline{a}_i, a_i] \quad (24)$$

Note that we will employ triangular approximation for calculation of fuzzy number in this paper, although the precise calculation has been obtained in Tanaka, Watada and Asai [5].

That is, the membership function $\mu_{L_o}(y)$ of the fuzzy grade of structural total damage $V_0$ can be obtained through extension principle to

$$\mu_{V_0}(y) = \bigvee_{(t_i, u_i) \in A_i} \left\{ \mu_{A_i}(t_i) \bigwedge \mu_{L_i}(u_i) \right\} \quad (25)$$

Table 2: Fuzzy data translated.

<table>
<thead>
<tr>
<th>Training Sample</th>
<th>Fuzzy grade of variables</th>
<th>Fuzzy grade of the objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$U_{L_1}(1)$ ··· $U_{L_1}(1)$ ··· $U_{L_K}(1)$</td>
<td>$V_{L_0}(1)$</td>
</tr>
<tr>
<td>2</td>
<td>$U_{L_1}(2)$ ··· $U_{L_1}(2)$ ··· $U_{L_K}(2)$</td>
<td>$V_{L_0}(2)$</td>
</tr>
<tr>
<td>3</td>
<td>(0.4,0.15,0.15) ··· (0.6,0.15,0.15) ··· (0.8,0.15,0.15)</td>
<td>(0.6,0.15,0.15)</td>
</tr>
<tr>
<td>···</td>
<td>···</td>
<td>···</td>
</tr>
<tr>
<td>ω</td>
<td>$U_{L_1}(\omega)$ ··· $U_{L_1}(\omega)$ ··· $U_{L_K}(\omega)$</td>
<td>$V_{L_0}(\omega)$</td>
</tr>
<tr>
<td>···</td>
<td>···</td>
<td>···</td>
</tr>
<tr>
<td>n</td>
<td>$U_{L_1}(n)$ ··· $U_{L_1}(n)$ ··· $U_{L_K}(n)$</td>
<td>$V_{L_0}(n)$</td>
</tr>
</tbody>
</table>
When the value $V_{L_0}(\omega)$ given Equation (21) is obtained, Equation (21) enables us to define its membership function using parameter $t_i$ for $A_i$ and parameter $u_i$ for $L_i$ of Equation (21).

(2) the fuzzy grade.
In this section, we discuss the heuristic method to determine a fuzzy assessment function by using non-fuzzy grades of assessment attributes, i.e., $U_i$ are fuzzy numbers in $[0,1]$.

According to the sign of $A_i$, the production of fuzzy number $A_i$ and $U_L$ is given in the following three cases:

(i) In this case where $\pi_i \geq \alpha_i \geq 0$

\[
(A_iU_L)^{h^0} = [a_i u_i, \alpha_i \pi_i]
\]

(ii) In this case where $\pi_i \leq \alpha_i \leq 0$

\[
(A_iU_L)^{h^0} = [a_i \pi_i, \alpha_i u_i]
\]

(iii) In this case where $\pi_i \leq \alpha_i \leq 0$

\[
(U_L)^{h^0} = [a_i \pi_i, \alpha_i u_i]
\]

It is difficult to solve analytically this problem. Therefore, we employ the heuristic approach for solving the problem. The procedure is as follows.

An $\alpha$-level set of the fuzzy degree of a structural attribute $U_L(i=1,2,\ldots,K)$ at $h^0$ is assumed to be denoted by Equation (24).

References


